Stabilizer Slicing: Coherent Error Cancellations in Low Density Parity Check Codes

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Coherent Errors in QEC

- Expect them to be worse than stochastic counterparts. (Knill, arXiv, 2004)
- Most research has focused on memory errors:

How to get around them:

- Dynamical Decoupling
  - Viola, Knill, Lloyd, **PRL**, 1999
- Random Compiling
  - Wallman, Emerson, **PRA**, 2016
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Help at high coherence but add some overhead.

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- Geller, Zhou, **PRA**, 2013
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Find that they are worse than stochastic, but not as bad as thought.

Understand how they interact with distance:

- Beale et al., **PRL**, 2018
- Huang, Doherty, and Flammia, **PRA**, 2019

Find that they approach stochastic as distance increases.

Statement of Problem

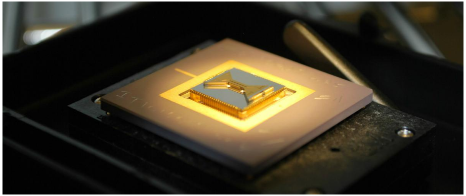
- In many quantum computing architectures, gate errors are the primary concern. As a result, the act of applying error correction circuits can cause additional error.
- Depending on the architecture, this error can come in the form of coherent overrotation, meaning it is caused by a unitary channel in the direction of the gate itself.

Our Solution

Use this structure and coherence to our advantage and eliminate these gate errors by directing them opposite each other.

Physical Motivation

- These coherent overrotation errors often arise from a classical miscalibration. In an ion-trap quantum computer this could be a miscalibration in laser intensity or timing.
- Even for architectures which also have issues with T_2 times, our technique will help with any coherence that does exist in the system.

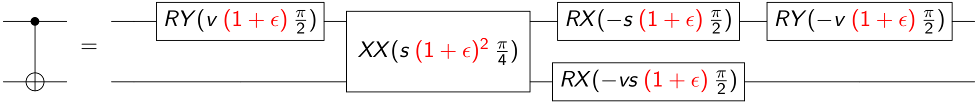


What is a Coherent Overrotation Error?

In the field of quantum error correction, coherent errors are considered to be more damaging that stochastic errors on since they have:

- non-zero off-diagonal terms in the process matrix.
- error probabilities which add quadratically.

However, unlike stochastic errors these coherent errors are unitary channels and therefore invertible.



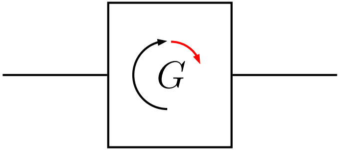
$$s, v = \pm 1$$

Maslov, **New J. Phys.**, 2017.

Error Model

- Errors follow every gate, and we assume that the strength of the overrotation is constant in time.
- Allow ourselves to vary strength of error and level of coherence.

Whenever a gate G is applied in our simulation, it is followed by an error of the form in Eq. 1:



$$\varepsilon_G(\rho) = \kappa \cdot \varepsilon_G^c(\rho) + (1-\kappa)\cdot \varepsilon_G^s(\rho) \tag{1}$$

$$\begin{aligned} \varepsilon_G^c(\rho) &= \exp(-i\epsilon G)\rho \exp(i\epsilon G) \\ \varepsilon_G^s(\rho) &= \cos^2(\epsilon) I\rho I + \sin^2(\epsilon) G\rho G. \end{aligned} \tag{2}$$

Wallman, Granade, Harper, Flammia, **New J. Phys.**, 2017. Trout et al., **New J. Phys.**, 2018

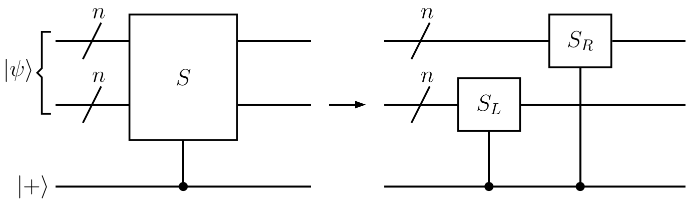
Architectural Requirements

To implement our method, an architecture will need to satisfy two conditions:

- The freedom to apply any rotational gate in the clockwise or counterclockwise direction.
- For a code with $2n$-body stabilizers, the ability to generate native multi-qubit gates by evolving an $(n+1)$-body Hamiltonian

The first condition is already satisfied for many architectures, and in an ion-trap quantum computer corresponds to being choose the direction of our Mølmer-Sørensen gates.

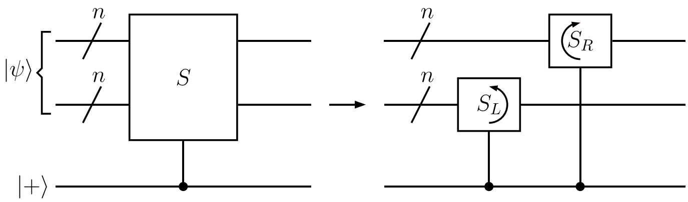
Stabilizer Slicing



Our Plan:

- Split the stabilizer S into two pieces S_L and S_R.

Stabilizer Slicing



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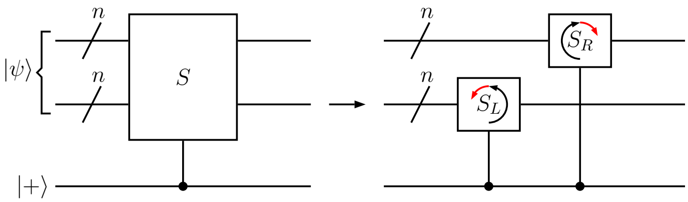
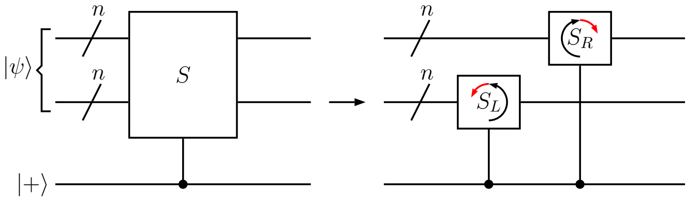


Our Plan:

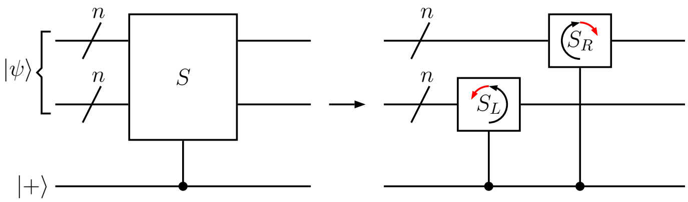
- Split the stabilizer S into two pieces S_L and S_R.
- Apply these gates as $\pm\pi$ rotations where the angles are chosen so the gates rotate in opposite directions.
- Due to the symmetries of our stabilizer state, the overrotation errors will then cancel out.

Stabilizer Slicing



$$\begin{aligned} S|\psi\rangle &= |\psi\rangle \\ S_R|\psi\rangle &= S_L|\psi\rangle \\ \therefore \exp(i\theta S_R)|\psi\rangle &= \exp(i\theta S_L)|\psi\rangle \end{aligned} \tag{3}$$

Stabilizer Slicing



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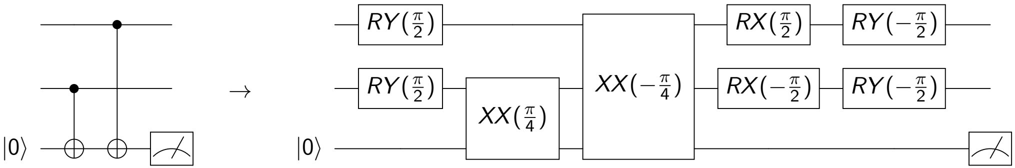
$$

$$
\begin{aligned}
U_E^L U_E^R|\psi\rangle|+\rangle &= \exp(i\epsilon\theta_L CS_L)\exp(i\epsilon\theta_R CS_R)|\psi\rangle|+\rangle \\
&= \exp(i\epsilon\pi CS_L)\exp(i\epsilon(-\pi)CS_L)|\psi\rangle|+\rangle \\
&= |\psi\rangle|+\rangle
\end{aligned} \tag{4}
$$

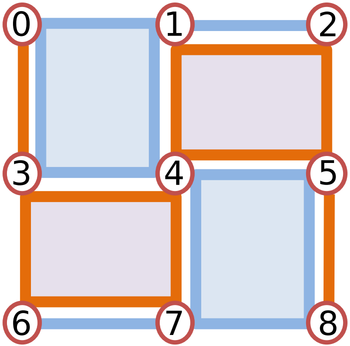
Ion Trap Example: 2 Qubit Stabilizer

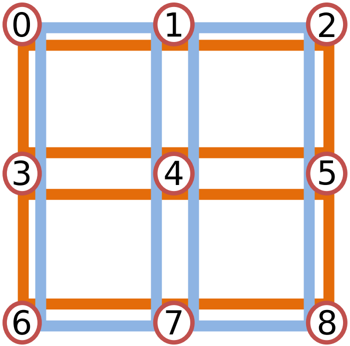


- I've chosen $s = -s' = 1$ to implement slicing, and $v = v' = 1$ to best cancel single qubit gates.
- In this weight-2 stabilizer, we only need weight-2 gates to properly slice it.

Codes Considered

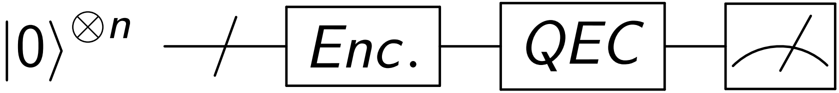
- We simulated our technique on the [[9,1,3]] Rotated Surface and Bacon-Shor codes.





Tomita, Svore, **PRA**, 2014. Bombín, Delgado, **PRA**, 2007. Aliferis, Cross **PRL**, 2007. Bacon, **PRA**, 2006. Li, **PRA**, 2018.

Testbed Circuit

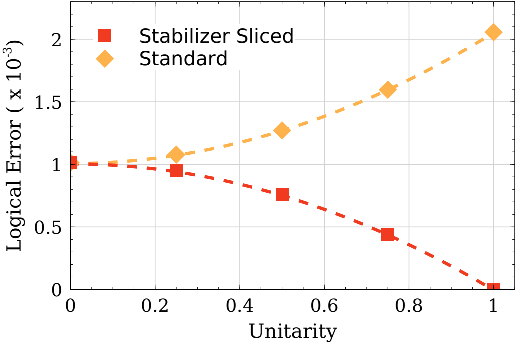


- Run QEC with circuit level error on a perfect encoded state.
- Measure all data qubits and classically decode to find logical error rate.
- Logical error rates are calculated using *quantumsim*, a full density-matrix simulator.

https://gitlab.com/quantumsim/quantumsim

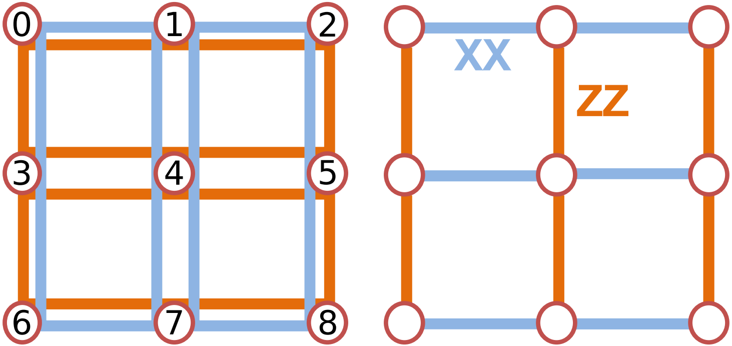
Surface-17: Simplified Model

- First we implement stabilizer slicing on a $[[9,1,3]]$ surface code where we have allowed ourselves 2 and 3 qubit Clifford operations.
- We only put errors on the entangling gates.
- Gate error rate of 10^{-3} on both weight-2 and weight-3 gates.



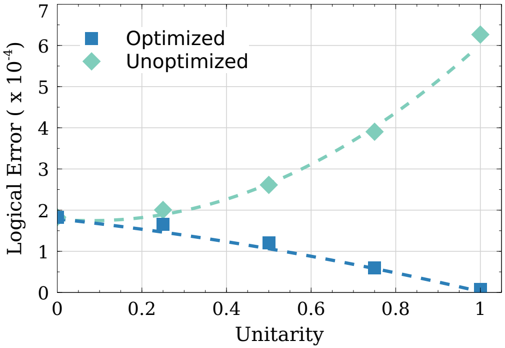
Gauges in Bacon-Shor-13

The Bacon-Shor code is a subsystem code, meaning that it has less than $n - k$ stabilizers. This means that there are **gauge operators** which commute with all the stabilizers but do not change the logical information of the encoded state.



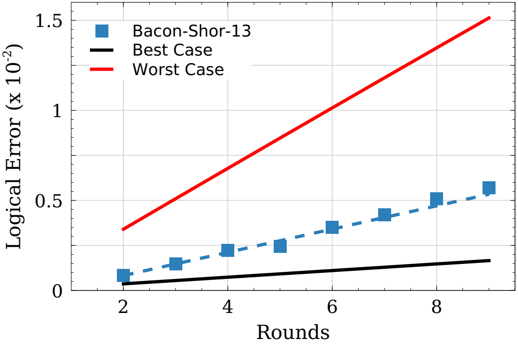
Bacon-Shor-13: Ion Trap Gate Set

- In this simulation we slice gauges on a $[[9, 1, 3]]$ Bacon-Shor code where the circuit has been decomposed into ion trap gates.
- There are errors on all gates, so we do not fully eliminate error.
- Gate error rate of 5×10^{-4} for single qubit gates.



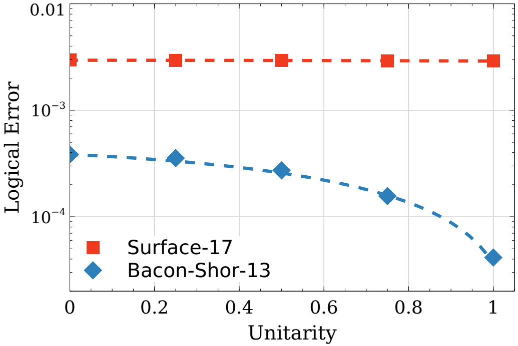
Effects of Gauge Flipping

- Errors and corrections in the Z-type stabilizer measurements will flip the gauges. This will cause our cancellations to disappear.
- By changing our circuit based on previous measurements we can follow the black line.
- Gate error of 10^{-3} and $\kappa = 1$.

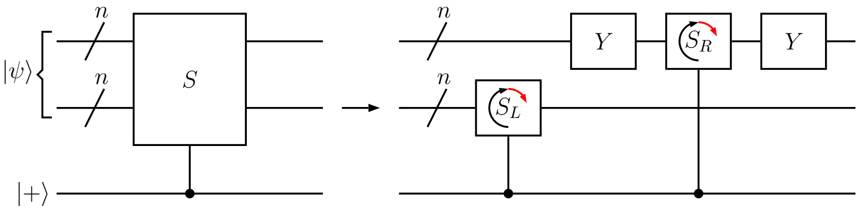


Fully Optimized Performance Ion Trap Gates and Stark Shift Errors

- Comparison of fully optimized and canceled circuits for Surface-17 and Bacon-Shor-13.
- Circuits are made with currently available ion trap gates.
- Also include Z-type Stark shift errors during gates to make model more realistic.
- Gate error rate of 10^{-3}.



Connections to Dynamical Decoupling



- If an architecture does not permit differently directed gates, these single qubit gates can be used to apply the same technique.
- Produces similar results depending on the quality of your single qubit gates.
- Other studies of the intersection of Dynamical Decoupling and QEC can be found in Ng, Lidar, Preskill (2011) and Cai, Xu, Benjamin (2019).

Stabilizer Slicing Conclusion

- Implementing stabilizer slicing significantly reduces the impact of overrotation error in the coherent case.
- There is no overhead in the way of additional gates, timing, or qubits.
- The technique can be applied to a wide range of codes.

More Information

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