Stabilizer Slicing: Coherent Error Cancellations in LDPC Codes

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Abstract

Coherent errors are a dominant noise process in many quantum computing architectures. Unlike stochastic errors, these errors can combine constructively and grow into highly detrimental overrotations. To combat this, we introduce a simple technique for suppressing systematic coherent errors in low-density parity-check (LDPC) stabilizer codes, which we call *stabilizer slicing*.
For conventional 2-body ion trap gates, we observe an 89-fold improvement for Bacon-Shor-13 with purely coherent errors which should be testable in near-term fault-tolerance experiments. With access to native gates generated by 3-body Hamiltonians, we can completely eliminate purely coherent overrotation errors, and for overrotation noise of 0.99 unitarity we achieve a 135-fold improvement in the logical error rate of Surface-17. The first scheme takes advantage of the prepared gauge degrees of freedom, and to our knowledge is the first example in which the state of the gauge directly affects the robustness of a code's memory. This work demonstrates that coherent noise is preferable to stochastic noise within certain code and gate implementations when the coherence is utilized effectively.

Architectural Requirements

To perform stabilizer slicing, we require a quantum computing architecture with two particular experimental degrees of freedom.

- Our architecture gives us the directional freedom to apply any gate in the clockwise or counterclockwise direction.
- For a code with $2n$-body stabilizers, our architecture can generate native multi-qubit gates by evolving an $(n+1)$-body Hamiltonian.

Error Model

We model our overrotation errors as a mixed channel with two parameters, as this is a better model of physical errors. The first parameter is the unitarity κ. The second parameter is the overrotation angle ϵ, which controls the strength of the error. Consequently, the error following some perfect gate G has the form,

$$\varepsilon_G(\rho) = \kappa \cdot \varepsilon_G^c(\rho) + (1-\kappa)\cdot \varepsilon_G^s(\rho)$$

where ε_G^c and ε_G^s are coherent and stochastic overrotation channels with equal fidelity given by,

$$\varepsilon_G^c(\rho) = \exp(-i\epsilon G)\rho\exp(i\epsilon G)$$
$$\varepsilon_G^s(\rho) = \cos^2(\epsilon) I\rho I + \sin^2(\epsilon) G\rho G.$$

Ordinarily, we would expect the latter channel, corresponding to $\kappa = 0$, to produce lower logical error rates due to the dropout of off-diagonal terms. To best display the improvements of stabilizer slicing, we do not include measurement error.

Stabilizer Slicing

For a given stabilizer S, let S_L and S_R be two disjoint Pauli operators such that $S_L S_R = S$. Then if $|\psi\rangle$ is a clean codestate,

$$S|\psi\rangle = |\psi\rangle$$
$$S_R|\psi\rangle = S_L|\psi\rangle$$
$$\therefore \exp(i\theta S_R)|\psi\rangle = \exp(i\theta S_L)|\psi\rangle.$$

If we apply our stabilizer through two controlled $\pi/2$-rotations with opposing directions and the error model above we have the following overrotation errors:

$$\begin{aligned} U_E^L U_E^R|\psi\rangle|+\rangle &= \exp(i\epsilon\theta_L CS_L)\exp(i\epsilon\theta_R CS_R)|\psi\rangle|+\rangle \\ &= \exp(i\epsilon\theta_L CS_L)\exp(i\epsilon(-\theta_L)CS_L)|\psi\rangle|+\rangle \\ &= |\psi\rangle|+\rangle. \end{aligned}$$

More Information and References

- **arXiv**: 1810.01040
- **Phys Rev Letters**: 121.250502

Codes Considered

Low Density Parity-Check (LDPC) Codes are quantum error correcting code families in which the number of stabilizers each qubit interacts with, and the number of qubits involved in each stabilizer, do not increase as the distance of the code is increased. The most popular example is the **rotated surface code** which we consider. We also have an application using the **Bacon-Shor** code, a quantum error correcting code family that is not LDPC but still has symmetries which we can use for our technique, at a reduced effectiveness (See Figures 2 and 3). In our simulations we consider the [[9,1,3]] versions of these two codes.

Results

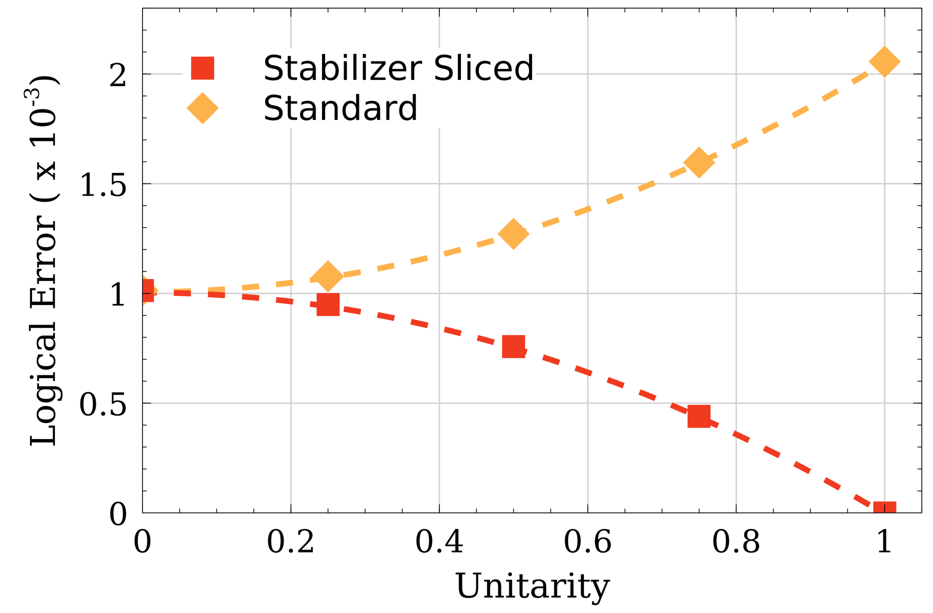

Figure 1: Logical error rates and quadratic fits for Surface-17 assuming access to native 3-body gates. As expected, in the fully coherent case we completely eliminate the noise present in our model. 2- and 3-qubit gate infidelities are 1.0×10^{-3}.

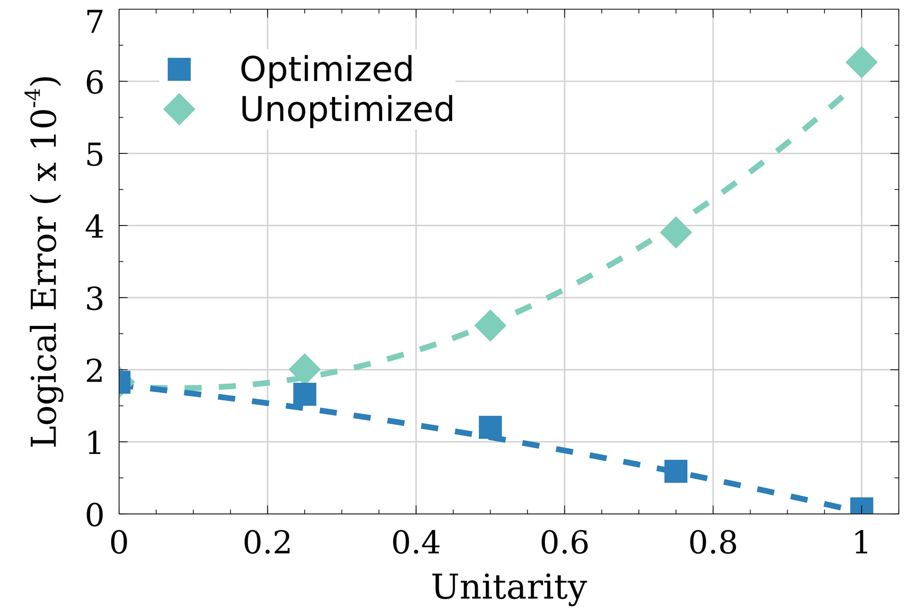

Figure 2: One-sided logical error rates for Bacon- Shor-13. The optimized line does not intersect the x-axis in the purely coherent case since we have included single qubit overrotations. Two-qubit gate infidelity is $\sin^2(\epsilon_2) = 5.0\times10^{-4}$ and $\epsilon_2 = (1+\epsilon_1)^2 - 1$.

Bacon-Shor Gauges

In Bacon-Shor codes, there are gauge operators which represent degrees of freedom which do not change the logical state and commute with all the stabilizers. These operators only have their eigenvalues changed due to errors and subsequent corrections. We prepare a state in which all of these gauge operators start with +1 eigenvalues, and then treat them as weight-2 stabilizers. As errors occur these eigenvalues flip sign and our cancellations disappear, leading to worsened performance. This is the first case in which the eigenvalues of gauge operators are shown to have an impact on the performance of a quantum memory.

Effects of Gauge Decay in [[9,1,3]] Bacon-Shor

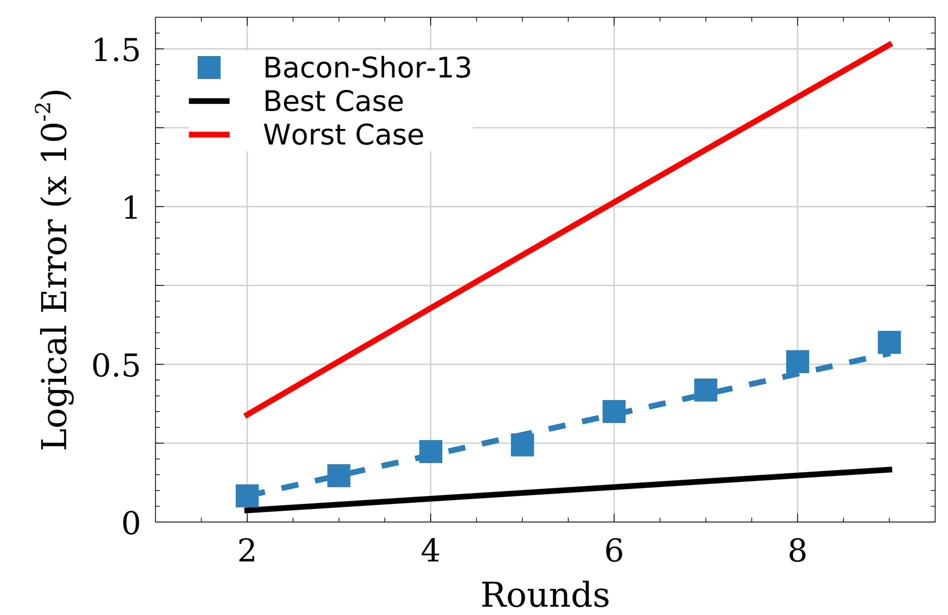

Figure 3: One-sided logical error rates from multiple rounds of error correction using the fully optimized Bacon Shor circuit in the purely coherent case of Figure 2's error model.

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